MSRI

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KAROL KOZIOL Email/Phone: kkoziol@ualberta.ca

Speaker's Name: ZHIWEI YUN

Talk Title: SPECIALIZATION TO THE DIAGONAL II

Date: 4 / 9 / 19 Time: 3 :30 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER CONTINUED TO DISCUSS THE SPECIALIZATION THEOREM FROM THE PREVIOUS LECTURE AND SHOWED HOW TO CONSTRUCT THE GALOIS ACTION ON STALKS OF CERTAIN IND-CONSTRUCTIBLE SHEAVES

CHECK LIST

(This is **NOT** optional, we will **not** pay for **incomplete** forms)

- ☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - Computer Presentations: Obtain a copy of their presentation
  - Overhead: Obtain a copy or use the originals and scan them
  - Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - Handouts: Obtain copies of and scan all handouts
- ☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

SPECIALIZATION TO THE DIAGONAL II - YUN

$$sp^* : \mathcal{H}_I(W)_{\Delta\bar{\eta}} \xrightarrow{\sim} \mathcal{H}_I(W)_{\overline{\eta^I}} = H_I(W)$$

THINK $\mathcal{H}_I(W)$ IS CONSTRUCTIBLE ON $U^I \subset X^I$

- IN CHAR 0, IRR. LOCAL SYSTEM ON $U \times U$

$$\Longleftrightarrow \text{REP OF } \pi_1(U \times U) = \pi_1(U) \times \pi_1(U)$$

$$\Longrightarrow \mathcal{L} \cong \mathcal{L}' \boxtimes \mathcal{L}''$$

- IN CHAR p, HAVE MAP

$$\pi_1(U \times U) \longrightarrow \pi_1(U) \times \pi_1(U),$$

NOT ISOM IN GENERAL

EX $U = \mathbb{A}^1$

$$\begin{array}{ccc} f^*AS_\psi & & AS_\psi \\ \mathbb{A}^1 \times \mathbb{A}^1 & \xrightarrow[f(x,y)]{} & \mathbb{A}^1 \end{array} \quad = \text{ARTIN} - \text{SCHREIER}$$

GENERIC

EG $f(x,y) = xy$

$$f^*AS_\psi \not\cong \mathcal{L}_1 \boxtimes \mathcal{L}_2$$

$$\boxed{\begin{array}{l} \text{THM } 1) \ \exists \ H_I(-) : \operatorname{REP}(\hat{G}^I) \longrightarrow \operatorname{REP}_{\text{CTS}}\big(\text{WEIL}(X,\bar{\eta})^I \times \mathbb{T}\big) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \| \qquad\qquad \mathbb{T} \subset C_c(G(\mathbb{O})\backslash G(\mathbb{A})/G(\mathbb{O})) \\ \qquad\qquad\qquad\qquad\qquad\qquad \left(\text{FINITE-TYPE } \mathbb{T} \text{ MOD W/ CTS WEIL GP ACTION}\right) \\ \left(\text{A PRIORI, } H_I(W) \text{ HAS AN ACTION OF } \operatorname{GAL}(\overline{\eta^I}/\eta^I) \times \mathbb{T}\right) \end{array}}$$

2) $\phi : J \longrightarrow I$, $W \in \mathrm{REP}(\widehat{G}^J)$

$$\rightsquigarrow \quad \widehat{G}^I \longrightarrow \widehat{G}^J$$

NOTATION:

$$H_I(W) \overset{\text{DEF}}{=} \mathcal{H}_I(W)\big|_{\Delta\bar{\eta}}$$

$$\cancel{H_J(W)\big|_{\Delta_\phi \bar{\eta}^J} = H_I(\mathrm{RES}_\phi(W))}$$

$$H_J(W) \underset{\text{AS HECKE MODULES}}{\overset{\text{FUSION}}{\cong}} H_I(\mathrm{RES}_\phi(W))$$

IS ISOM IS EQUIV'T AS $\mathrm{WEIL}(X,\bar{\eta})^I$-MODULES, WHERE LHS USES $\mathrm{WEIL}^I \xrightarrow[\Delta_\phi]{} \mathrm{WEIL}^J$

$$\mathrm{REP}(\mathrm{WEIL}^I) \longleftarrow \mathrm{REP}(\mathrm{WEIL}^J) : \mathrm{RES}_\phi$$

SO 2) SAYS

$$\mathrm{RES}_\phi(H_J(W)) \cong H_I(\mathrm{RES}_\phi(W)) \in \mathrm{REP}(\mathrm{WEIL}^I \times \Pi)$$

WHAT IS ACTING ON $H_I(W) \cong \mathcal{H}_I(W)\big|_{\Delta\bar{\eta}} \cong \mathcal{H}_I(W)\big|_{\bar{\eta}^I}$?

- $\mathrm{GAL}(\bar{\eta}^I/\eta^I) = \mathrm{GAL}(F_I^{\mathrm{SEP}}/F_I)$ IS ACTING
- PARTIAL FROB

$$F_i : \mathrm{FROB}_i^* H_I(W) \longrightarrow H_I(W) \quad \text{AS SHEAVES ON } \eta^I$$

TRANSLATE THESE STRUCTURES INTO AN ACTION OF A BIGGER GRP

FIX $\overline{\eta^I} \rightsquigarrow \Delta\overline{\eta}$

$$F \leftarrow \overline{F}_1 \otimes_{\overline{k}} \cdots \otimes_{\overline{k}} \overline{F}_n \longrightarrow \overline{F}_I \qquad (\text{ALL } \overline{F}_i \text{ ISOM'C})$$

$$\mathrm{FWEIL}(\eta^I, \overline{\eta^I}) = \left\{ \delta \in \mathrm{AUT}_{\overline{k}}(\overline{F}_I) : \begin{array}{l} \delta|_{F_i^{\mathrm{PERF}}} = \mathrm{FROB}^{n_i} \\ \text{FOR SOME } n_i \in \mathbb{Z} \\ \forall i \in I \end{array} \right\}$$

EX $I = \{1\}$

$$\mathrm{FWEIL}(\eta, \overline{\eta}) \xrightarrow{\sim} \mathrm{WEIL}(\overline{\eta}/\eta)$$

$$\delta \longmapsto \delta \circ \mathrm{FROB}^{-d(\delta)} \in \mathrm{AUT}_F(\overline{F}) = \mathrm{GAL}(F^s/F)$$

$$\left(\delta|_{F_i^{\mathrm{PERF}}} = \mathrm{FROB}^{b(\delta)} \right.$$

IN GENERAL

$$1 \longrightarrow \underset{\substack{\| \\ \pi_1^{\mathrm{GEOM}}(\eta^I, \overline{\eta^I})}}{\mathrm{GAL}(\overline{F}_I / F_I \otimes \overline{k})} \to \mathrm{FWEIL}(\eta^I, \overline{\eta^I}) \longrightarrow \mathbb{Z}^I \longrightarrow 0$$

$$\Longrightarrow \mathrm{FWEIL}(\eta^I, \overline{\eta^I}) \curvearrowright H_I(W) \curvearrowleft \mathbb{T}$$

$$pr : \text{FWEIL}(y^I, \bar{y}^I) \twoheadrightarrow \text{FWEIL}(y, \bar{y})^I$$
$$\| $$
$$\text{WEIL}(\bar{y}/y)^I$$

$$\delta \longmapsto (\delta|_{\bar{F}_i})_{i \in I}$$

<u>WANT TO SHOW</u> $\text{FWEIL}(y^I, \bar{y}^I) \circlearrowright H_I(W)$ FACTORS THROUGH $\text{WEIL}(\bar{y}/y)^I$

KEY: pr INDUCES AN ISOM AFTER PASSING TO PROFINITE COMPLETION

$\Longrightarrow$ (PRETENDING $H_I(W)$ IS F.D. $/E$)

$H_I(W) \in \text{REP}_{\text{CTS}}(\text{WEIL}(\bar{y}/y)^I \times \mathbb{T})$

MODULAR CURVE CASE: $I = \{1\}$, $W =$ STD REP OF GL_2

$\text{GAL}(\bar{\mathbb{Q}}/\mathbb{Q}) \circlearrowright H^1(\text{MODULAR CURVE}, \bar{\mathbb{Q}}_\ell) \circlearrowleft \mathbb{T}$

MORE LEGS = MORE DIFFICULTIES

LOCAL ANALOG OF $\text{FWEIL}(y^I, \bar{y}^I)$

$I = \{1,2\}$ $\quad F_I = \text{FRAC}(F \otimes_k F)$ SYMMETRIC

LOCAL ANALOG:

$$k((x))((y)) \neq k((y))((x))$$

CHOOSE AN ORDERING ON I, $\alpha : I \xrightarrow{\sim} \{1, \dots, n\}$

$v \in X(k)$

F_v = COMPLETED LOCAL FIELD AT v $\cong k((t))$

$$F^\alpha_{I,v} = k((t_1))((t_2))\dots((t_n)), \quad \eta^\alpha_{I,v} = \mathrm{SPEC}(F^\alpha_{I,v})$$

CHOOSE ALG CLOSURE $\overline{F^\alpha_{I,v}} \supset \bar{k}$

$$\mathrm{FWEIL}(\eta^\alpha_{I,v}, \overline{\eta^\alpha_{I,v}}) = \left\{ \delta \in \mathrm{AUT}_k(\overline{F^\alpha_{I,v}}) : \delta(t_i) = t_i^{q^{n_i}}, \ n_i \in \mathbb{Z} \ \forall i \in I \right\}$$

= WEIL GP IF $|I| = 1$

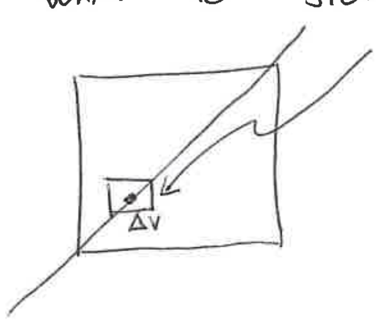
$$pr^\alpha : \mathrm{FWEIL}(\eta^\alpha_{I,v}, \overline{\eta^\alpha_{I,v}}) \longrightarrow \mathrm{WEIL}(\bar{\eta}_v/\eta_v)^I$$

(CHOOSE $\overline{F_v}$ FOR i^{TH} FACTOR, IE $\overline{k((t_i))} \subset \overline{F^\alpha_{I,v}}$)

__FACT__ pr^α IS SURJECTIVE

__LOCAL - GLOBAL COMPAT.__

WANT TO STUDY



FOR $I = \{1\}$: $\mathcal{F}$ SHEAF ON X

CHOOSE $\bar{\eta} \rightsquigarrow \bar{v}$. WE GET

$$sp^*: \mathcal{F}_{\bar{v}} \longrightarrow \mathcal{F}_{\bar{\eta}}$$

$\circlearrowleft$ $\mathrm{GAL}(\bar{v}/v)$ $\qquad$ $\circlearrowleft$ $\mathrm{GAL}(\bar{\eta}/\eta)$

BUT NO MAP B/W GALOIS GPS

$$\begin{tikzcd}[column sep=small]
\rightsquigarrow \quad \mathrm{Im}(sp^*) & \subset & \mathcal{F}_{\bar\eta}^{I_v} & \subset & \mathcal{F}_{\bar\eta} \\
 & & \circlearrowleft \ \mathrm{Gal}(\bar v/v) & & \\
\mathcal{F}_{\bar v} \ \circlearrowleft \ \mathrm{Gal}(\bar v/v) \arrow[uu, two heads] \arrow[uurr] & & & &
\end{tikzcd}$$

(AFTER CHOOSING DECOMP GP)

THIS MAP IS EQUIV'T

<u>FACT</u> FIX

$$\begin{tikzcd}
\overline{F}_I \arrow[r, hook] & \overline{F_{I,v}^{\alpha}} \\
\overline{F}\otimes_{\bar k}\cdots\otimes_{\bar k}\overline{F} \arrow[u, phantom, "\cup"] \arrow[r, hook] \arrow[ur, phantom, "\circlearrowright"] & \overline{F}_v\otimes_{\bar k}\cdots\otimes_{\bar k}\overline{F}_v \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

GIVES $\overline{\eta^{\alpha}_{I,v}} \rightsquigarrow \Delta\bar v$

$$\Longrightarrow \quad \begin{tikzcd}
\mathrm{FWeil}(\eta^{\alpha}_{I,v}, \overline{\eta^{\alpha}_{I,v}}) \arrow[r, "i"] \arrow[d] \arrow[dr, phantom, "\circlearrowright"] & \mathrm{FWeil}(\eta^{I}, \overline{\eta^{I}}) \arrow[d] \\
\mathrm{Weil}(\bar\eta_v/\eta_v)^I \arrow[r, "i^I"] \arrow[dr] & \mathrm{Weil}(\bar\eta/\eta)^I \arrow[d] \\
 & \mathbb{Z}^I
\end{tikzcd}$$

THE FOLLOWING DIAGRAM COMMUTES

$$\begin{tikzcd}
\mathcal{H}_I(W)\big|_{\Delta\bar v} \arrow[r, "sp^*"] & \mathcal{H}_I(W)_{\overline{\eta^{\alpha}_{I,v}}} \arrow[r, "\sim"] & \mathcal{H}_I(W)_{\overline{\eta^{I}}} \cong H_I(W) \\
\circlearrowleft & \circlearrowleft & \circlearrowleft \\
\mathbb{Z}^I & \mathrm{FWeil}(\eta^{\alpha}_{I,v}, \overline{\eta^{\alpha}_{I,v}}) \arrow[l] \arrow[r, "i"] & \mathrm{FWeil}(\eta^{I}, \overline{\eta^{I}}) \arrow[d, "\text{ACTION FACTORS}"] \\
 & & \mathrm{Weil}(\bar\eta/\eta)^I
\end{tikzcd}$$

BY PARTIAL FROB

$$(d_i(\delta))_{i\in I} \longleftarrow\!\shortmid \ \delta \ \longmapsto i(\delta)$$