MSRI
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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Karol Koziol Email/Phone: kkozol@ualberta.ca

Speaker's Name: Xinwen Zhu

Talk Title: Local Langlands Parametrization and Local - Global Compatibility I

Date: 4 / 12 / 19 Time: 9 : 30 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: The speaker presented a thm about the identification of certain excursion operators as Hecke operators. Parts of the proof were discussed, making use of cohomological correspondences

CHECK LIST

(This is **NOT** optional, we will **not** pay for **incomplete** forms)

☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- Computer Presentations: Obtain a copy of their presentation
- Overhead: Obtain a copy or use the originals and scan them
- Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- Handouts: Obtain copies of and scan all handouts

☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

LOCAL LANGLANDS PARAMETRIZATION + LOCAL/GLOBAL COMPATIBILITY

— ZHU

TO DO

- E–S RELN
- S = T
- CONSTRUCTION OF LOCAL EXCURSION ALG
- L/G C

NOTATION

- X/k CURVE
- $v \in |X|$, $\deg(v) = 1$
- WILL CONSIDER SHTUKA W/ LEVEL nv
- $\eta \in X$, $\eta^I \in X^I$
- $W \in \mathrm{Rep}(\hat{G}^I)$ $\rightsquigarrow$ $x : \mathbb{1} \longrightarrow \mathrm{Res}_\phi(W)$

  $\xi : \mathrm{Res}_\phi(W) \longrightarrow \mathbb{1}$

- $\alpha : I \xrightarrow{\sim} \{1, \dots, n\}$

  $\eta_v^\alpha \cong k((\varpi_1)) \dots ((\varpi_n))$

- $\Delta_{\underline{d}} = \mathrm{FROB}^{\underline{d}} \circ \Delta : X \longrightarrow X^I$

  $\underline{d} : I \longrightarrow \mathbb{Z}_{\geq 0}$

- $\overline{\eta} \longrightarrow \eta$, $\quad \overline{v} \longrightarrow v$, $\quad \overline{\eta} \rightsquigarrow \overline{v}$

$$
\begin{tikzcd}
\overline{\eta}^{\alpha}_{v} \arrow[r] \arrow[d] & \overline{\eta^{I}} \arrow[r, squiggly] \arrow[d] & \Delta(\overline{\eta}) \\
\eta^{\alpha}_{v} \arrow[r] & \eta^{I} &
\end{tikzcd}
$$

- $\delta \in \mathrm{FWEIL}(\eta^{\alpha}_{v}, \overline{\eta^{\alpha}_{v}}) \longrightarrow \mathrm{FWEIL}(\eta^{I}, \overline{\eta^{I}})$

HAVE COMM. DIAG.

$$
\begin{tikzcd}
\overline{\eta}^{\alpha}_{v} \arrow[r, "\delta"] \arrow[d] & \overline{\eta}^{\alpha}_{v} \arrow[d] \\
\overline{\eta^{I}} \arrow[r, "\delta"] \arrow[d] & \overline{\eta^{I}} \arrow[d] \\
\eta^{I} \arrow[r, "\mathrm{FROB}^{d}"] & \eta^{I}
\end{tikzcd}
$$

-
$$
\begin{tikzcd}
\mathcal{H}_{\{x\}}(\mathbb{1}) \arrow[r, "x"] \arrow[d, "S_{n,I,x,\xi,\delta}"] & \mathcal{H}_{\{x\}}(W)\big|_{\Delta(\overline{\eta})} \arrow[r, "\sim"] & \mathcal{H}_{I}(W)_{\overline{\eta}^{\alpha}_{v}} \arrow[d, "\delta"] & = \delta^{*}\mathcal{H}_{I}(W)_{\overline{\eta}^{\alpha}_{v}} \arrow[d, "\wr"] \\
 & & & \big((\mathrm{FROB}^{d})^{*}\mathcal{H}_{I}(W)\big)\big|_{\overline{\eta}^{\alpha}_{v}} \\
\mathcal{H}_{\{x\}}(\mathbb{1}) & \mathcal{H}_{\{x\}}(W)\big|_{\Delta(\overline{\eta})} \arrow[l, "\xi"'] \arrow[r, "\sim"] & \mathcal{H}_{I}(W)_{\overline{\eta}^{\alpha}_{v}} & \arrow[l, "\sim"']
\end{tikzcd}
$$

DEFINES EXCURSION OPERATOR

③

THM 1) $\exists$ A CANONICAL ELEMENT $z_{n,I,x,\xi,\delta}$

$\in Z(C_c(K_n \backslash G(F_v)/K_n, \Lambda))$ SUCH THAT

$S_{n,I,x,\xi,\delta} = z_{n,I,x,\xi,\delta}$ AS OPERATORS

ON $\mathcal{H}_{\{x\}}(\mathbb{1}) = C_c(\Xi G(F) \backslash G(\mathbb{A}) / K^v K_n)$

2) $n = 0$, $I = \{1, 2\}$, $\delta = (\text{FROB}, 1)$

$x : \mathbb{1} \xrightarrow{\text{COEV}} V \otimes V^*$, $\xi : V^* \otimes V \xrightarrow{\text{EV}} \mathbb{1}$

$\Rightarrow z_{0,\{1,2\},x,\xi,\delta} = h_{V,v}$

3) $z_{n,I,x,\xi,\delta}$ DEPENDS ONLY ON G_{F_v}

4) $z_{n,I,x,\xi,\delta}$ COMPATIBLE WITH EACH OTHER AS

n VARIES

$\left(n > n' : Z(C_c(K_n \backslash G(F_v)/K_n)) \xrightarrow{* 1_{K_{n'}}} Z(C_c(K_{n'} \backslash G(F_v)/K_{n'})) \right)$

ASIDE: $\exists\, X/k$ SMOOTH PROJ w/ $\pi_1(X) \longrightarrow \Gamma$

ORD 168

AND $\exists\ \Gamma \overset{\rho_1}{\underset{\rho_2}{\rightrightarrows}} SO(8)$

EVERYWHERE UNRAM'D

w/ $\rho_1(\gamma) \sim \rho_2(\gamma)$ $\forall \gamma \in \Gamma$

BUT $\rho_1 \not\sim \rho_2$

④

$\Pi \subset C_c(G(F)\backslash G(\mathbb{A})/G(\mathbb{O}))$ $\qquad G = SO(8)$

$\mathcal{B} \subset$

AND $\mathcal{B}$ SHOULD BE QUADRATIC $/\ \Pi$,

AND $\exists$ $S_{pf, \gamma_1, \ldots, \gamma_4} \notin \Pi$

$\left(\gamma_i \in \Gamma, \ pf \in E[\hat{G}^4]^{\hat{G}}\right.$

SHOULD DISTINGUISH ρ_1 AND ρ_2

Q WHAT IS IT?

NB EXCURSION ALG = HECKE ALG FOR GL_n, U_n, Sp_{2n}

PF

$$\begin{array}{ccccc} Sht_{I,nv,(W)} & & & & Sht_v \\ \downarrow & & & & \| \\ Sht_{I(W)}\big|_{(X-v)^I} & \hookrightarrow & Sht_{I,W} & \hookleftarrow & Sht_W\big|_{\Delta(v)} \end{array}$$

$$\psi^{NAIVE}_{\bar{t}}(W,n) := \psi^{NAIVE}_{\bar{t}\to\Delta(\bar{v})}\left(\varepsilon^* SAT_I(W) \otimes \pi_{n,*}\Lambda\right)$$

$\bar{t} \rightsquigarrow \Delta(\bar{v})$

$t \in (X-v)^I$

$$R\Gamma_c(Sht_v, \psi_{\bar{t}}(W,n)) \longrightarrow R\Gamma_c(Sht, \psi^{NAIVE}) \longrightarrow \mathcal{H}_I(W)\big|_{\bar{t}}$$

COMPLEX OF SHEAVES

$\left(\varepsilon : Sht_{v,W} \longrightarrow [G_{n,v}\backslash Gr_{v,W}] \quad \text{LOCAL MODEL}\right)$

π_n EXTENDS TO X^I IN TWO CASES

1) $W = \mathbb{1}$

$$\begin{tikzcd} G(F)\backslash G(\mathbb{A}) / K^v K_n \times X^I \arrow[d] \\ G(F)\backslash G(\mathbb{A}) / K^v G(\mathcal{O}_v) \times X^I \end{tikzcd}$$

2) $n = 0 \implies \varepsilon^* \mathrm{SAT}_I(W)$ is ULA (UNIVERSALLY LOCALLY ACYCLIC)

$$\implies \Psi_{\bar{\varepsilon}}(W, 0) \cong \varepsilon^* \mathrm{SAT}_{\bar{v}}(W) =: \mathcal{F}_W$$

CONSIDER THE CASE $\delta \longmapsto (\mathrm{FROB}, 1)$, $I = \{1,2\}$, $W = V \boxtimes V^*$

$$\begin{tikzcd} & \mathrm{Sht}^{\mathrm{LEFT}}_{\{1,2\}} \arrow[d, "m"] & & \mathrm{Sht}^{\mathrm{RIGHT}}_{\{1,2\}} \arrow[ll, "\mathrm{FROB}_1"'] \arrow[d, "m"] & \\ \mathrm{Sht}_{\bar{v},1} \arrow[r, hook] & \mathrm{Sht}_{\bar{v}} \arrow[dr] & & \mathrm{Sht}_{\bar{v}} \arrow[dl] & \mathrm{Sht}_{\bar{v},1} \arrow[l, hook'] \\ & & \bar{v} & & \end{tikzcd}$$

$$\begin{tikzcd} \mathcal{F}_{\mathbb{1}} = \varepsilon^* \mathrm{SAT}_{\bar{v}}(\mathbb{1}) \arrow[r] & \mathcal{F}_W = \varepsilon^* \mathrm{SAT}_{\bar{v}}(W) \\ & m_* \mathcal{F}_{V \tilde{\times} V^*} \arrow[u, "\wr"] \end{tikzcd}$$

$$\mathrm{FROB}^* \mathcal{F}_{V \tilde{\times} V^*} \cong \mathcal{F}_{V^* \tilde{\times} V}$$

ALSO HAVE DIAGRAM

$$\begin{tikzcd} m_* \mathcal{F}_{V^* \tilde{\times} V} \arrow[d, "\wr"] & \\ \mathcal{F}_W \arrow[r] & \mathcal{F}_1 \end{tikzcd}$$

COHOMOLOGICAL CORRESPONDENCES

$$\begin{tikzcd} & C \arrow[dl, "\overleftarrow{c}"'] \arrow[dr, "\overrightarrow{c}"] & \\ X & & Y \end{tikzcd}$$

SHEAVES $\mathcal{F}$ ON X
$\mathcal{G}$ ON Y

$$\boxed{\text{A } \underline{\text{COH'L CORR}} \text{ IS A MAP } \quad \overleftarrow{c}^* \mathcal{F} \longrightarrow \overrightarrow{c}^! \mathcal{G}}$$

SUPPOSE

$$\begin{tikzcd} & C \arrow[dl, "\overleftarrow{c}"'] \arrow[dr, "\overrightarrow{c}"] & \\ X \arrow[dr, "f"'] & & Y \arrow[dl, "g"] \\ & S & \end{tikzcd}$$

IF $\overleftarrow{c}$ IS PROPER (SCHEMATIC), A COH CORR

$$\overleftarrow{c}^* \mathcal{F} \longrightarrow \overrightarrow{c}^! \mathcal{G}$$

INDUCES

$$f_! \mathcal{F} \longrightarrow g_! \mathcal{G}$$

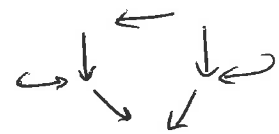
(REASON: $\overleftarrow{c}^* \mathcal{F} \to \overrightarrow{c}^! \mathcal{G} \implies \mathcal{F} \to \overleftarrow{c}_* \overrightarrow{c}^! \mathcal{G}$
$\implies f_! \mathcal{F} \to f_! \overleftarrow{c}_* \overrightarrow{c}^! \mathcal{G}$

+ USE PROPERNESS)

- CAN COMPOSE COH CORR.
- CAN PUSHFORWARD
- CAN PULLBACK IN CERTAIN CASES

CAN USE THIS TO GET COH CORR OF THE FORM

$$\begin{tikzcd}[column sep=small]
& G(F)\backslash G(\mathbb{A})/K \times^{G(\mathcal{O}_v)} G(F_v)/G(\mathcal{O}_v) \arrow[dl] \arrow[dr] & \\
G(F)\backslash G(\mathbb{A})/K^v G(\mathcal{O}_v) & & G(F)\backslash G(\mathbb{A})/K^v G(\mathcal{O}_v)
\end{tikzcd}$$

AND FORM PULLBACK SQUARE IN  DIAGRAM