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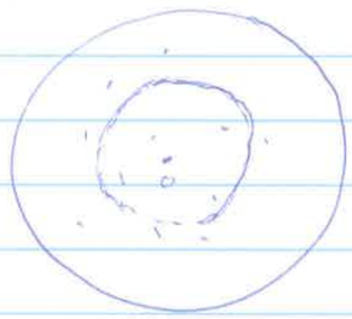
X Hadamard

Λ discrete group of isometries of X.

$$h_X(\Lambda) = \lim_{t\to\infty} \frac{\log \# \Lambda.o \cap B(o,t)}{t}$$

critical pt of Λ.



Counting how many pts of the orbits in $B_r(o)$ — Ball center at o with radius r.

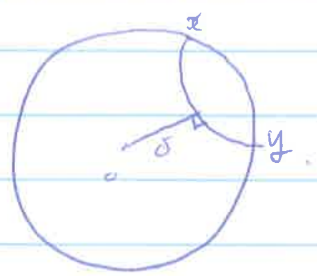
$K_X \leq -1$

Λ acts co-compactly (on a Λ invariant closed convex set)

$$h_X(\Lambda) = \lim_{t\to\infty} \frac{1}{t} \log \# \{ [g] \in [\Lambda] \mid |g| \leq t \}$$ where $|g|$ = translation dist of g = length of closed geod on $\Lambda \backslash X$

$h_X(\Lambda)$ is the topo entropy of geod flow on $\Lambda \backslash X$

$\partial_\infty X$ = visual boundary.



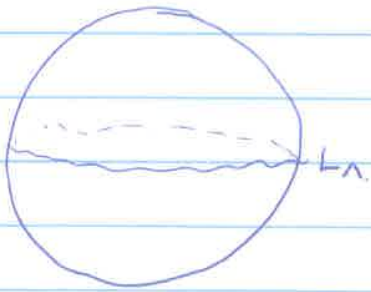
$e^{-\delta} = d_o(x,y) \rightsquigarrow$ visual metric on $\partial_\infty X$.

$\Lambda \curvearrowright \partial_\infty X$ has a smallest closed invariant set $= L_\Lambda$ limit set

Sullivan:

$$h_X(\Lambda) = \text{Hausdorff dim } (L_\Lambda)$$
$$\geq \text{topo dim } (L_\Lambda)$$

Λ = quasi-fuchsian group on $\mathbb{H}^3$.



$h_X(\Lambda) \geq 1$

Bowen. If $h_X(\Lambda) = 1$

$\Rightarrow \Lambda$ Fuchsian (i.e. preserves a totally geodesic copy of $\mathbb{H}^2$)

Beardon

Goal? Critical pts of h_X reveal geometric feature of the action $\Lambda \curvearrowright X$.

On QF space, there are $\underline{\infty}$ local maximal (Bridgeman)

• $G = SL(d, \mathbb{R})$ $X = G/K$ G's symmetric space

• Bishop – Steger: $\rho, \eta \in \mathcal{T}eich(S)$

$h(\rho, \eta) = \lim_{t \to \infty} \frac{1}{t} \log \# \{ \gamma : |\gamma|_\rho + |\gamma|_\eta \leq t \} \leq \frac{1}{2}$ " $= \frac{1}{2}$ " $\Rightarrow$ " $\rho = \eta$ "

• Burger: relate to $PSL(2,\mathbb{R}) \times PSL(2,\mathbb{R})$

• Crampon: M^n strictly convex projective manifold. (Hilbert metric geod flow)

h_{top} (geod flow) $\leq n-1$ " $= n-1$ " $\Rightarrow$ " M hyp "

$SL(2,\mathbb{R}) \longrightarrow SL(d,\mathbb{R})$! irred. rep (up to conjugacy)

$\pi_1(\Sigma) \xrightarrow{\rho_0} PSL(2,\mathbb{R}) \xrightarrow{\tau} PSL(d,\mathbb{R})$ Fuchsian rep $\tau \circ \rho_0$ where $\rho \in \mathcal{T}(\Sigma)$

Hitchin rep $:= \{\rho : \pi_1(\Sigma) \to PSL(d,\mathbb{R})\}$ can be deformed to a Fuchsian rep.

Thm (Potrie – S)

Let ρ be ~~in~~ the Hitchin component $\Rightarrow h_X(\rho) \leq 1$.

Moreover $h_X(\rho) = 1 \Rightarrow \rho$ is Fuchsian

- Zhang: $\exists$ sequences in Hitchin component such that $h_X(\rho_n) \to 0$ $(n \to \infty)$.

$|g|$

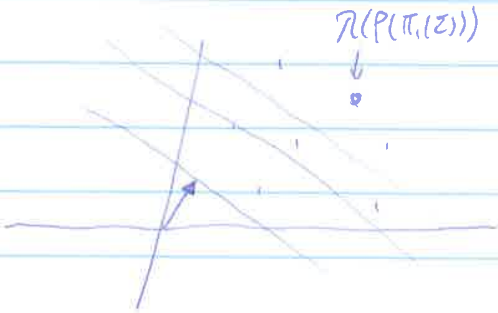
for $g \in SL(d, \mathbb{R})$ denote by

$\lambda(g) = (\lambda_1(g), \dots, \lambda_d(g))$ ~~by (eigenvalue)~~

$\lambda_i(g) =$ log of i-th eigenvalue of g.

$\mathfrak{a}^+ = \{(a_1, \dots, a_d) \mid a_1 + \dots + a_d = 0,\ a_1 \geq \dots \geq a_d\}$

$\rho \in$ Hitchin $(PSL(d, \mathbb{R}))$



count how many lies on that direction

$\varphi \in \mathfrak{a}^*$

$$\lim_{t \to \infty} \frac{\log \#\{\gamma \mid \varphi(\lambda(\rho\gamma)) \leq t\}}{t} = h^{\varphi}(\rho)$$

$h(\rho)$ can make sense if φ is positive on the closed cone generated by $\{\lambda(\rho(\gamma)),\ \gamma \in \pi_1(\Sigma)\}$

~~$k\varphi$~~

$h^{k\varphi} = \frac{1}{k} h^{\varphi}$

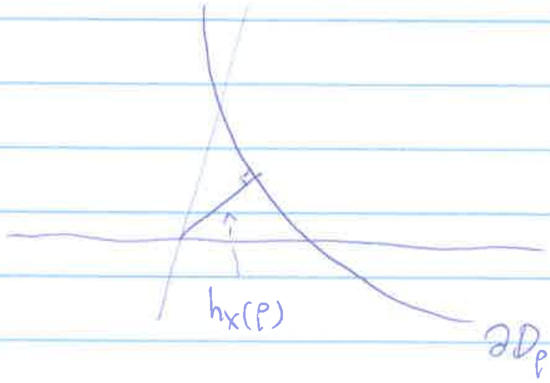
given φ the linear form

$h^{\varphi}\varphi$ has entropy 1

$\{\varphi : h^\varphi \in [0,1]\} = D_\rho$

$\{\varphi : h^\varphi = 1\}$ is boundary of convex set on $\mathfrak{a}^*$

$\inf \{ \|\varphi_0\|,\ \varphi_0 \in \{\varphi : h^\varphi = 1\} \} = h_X(\rho)$.



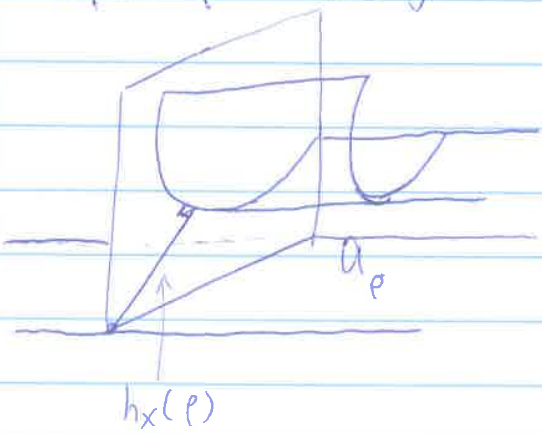
Quint.

$\partial D_\rho = \{\varphi : h^\varphi = 1\}$ is a closed

and analytic submanifold.

$\mathfrak{a}_\rho$ = vector space spanned by $\{\lambda(\rho\gamma) : \gamma \in \pi_1 \Sigma\}$

$D_\rho \cap \mathfrak{a}_\rho$ = strictly convex.



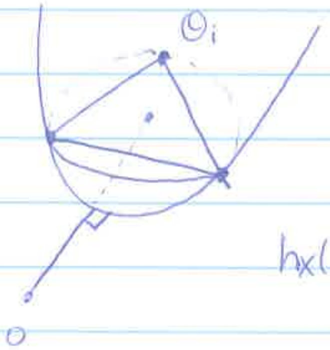
Thm (Potrie – S.)

Consider $\Theta_i(a_1, \ldots, a_d) = a_i - a_{i+1}$ (simple root).

$\Rightarrow h^{\Theta_i}(\rho) = 1 \quad \forall \rho$.

Inspired by a result by Deroin – Tholozan $\rho, \eta \in$ Hitch

$$\Rightarrow \sup_{\gamma \in \pi_1(\Sigma)} \frac{\Theta_i(\rho\gamma)}{\Theta_i(\eta\gamma)} \geq 1$$



$h_x(\rho) \leq \inf \{ \|\varphi\| : \varphi \in$ affine hyperplane generated by $\{\Theta_i\} \}$

equality $\Rightarrow$ ∂D_ρ intersects the interior of the simplex $\{\Theta_i\}$.

$\Rightarrow$ ∂D_ρ = affine hyperplane.

$\partial D_\rho \cap \mathfrak{a}_\rho$ is a point. (by convexity of the set)

$\Rightarrow$ $\mathfrak{a}_\rho$ is one dimensional $\Rightarrow$ ρ be rank 1 $\Leftrightarrow$ Fuchsian.

- entropy on some directions recognize rank of Zariski closure.

Ex: if $h^{t\Theta_2 + (1-t)\Theta_3} = 1 \Rightarrow \rho = Sp(4)$