1. Global rigidity of Anosov actions by higher rank lattices

Joint work with Brown, Rodriguez Hertz.

G semisimple without rank 0 or 1 factors.
$\Gamma \subset G$ is a lattice, Margulis superrigidity says that $\Gamma \to GL(V)$ must pass through G.

Zimmer's program: how about $\alpha : \Gamma \to Diff(M)$?

$f : M \to M$ is called Anosov if TM splits into $E^s \oplus E^u$. It is conjectured by Frantz that this only happens when M is infronil (covered by nilpotent).

Theorem: If Γ acts on infronil manifold M smoothly and as anosov maps, and the action lifts to its universal cover N, then the action passes through $Aut(M)$ after conjugation by a smooth diffeomorphism of M. (extends previous works by Margulis, Qian, Fisher, Katok, Luis, Zimmer)

Theorem A: If Γ-action is C^1 and lifts to N, $\exists \gamma_0$ whose action is hyperbolic on $\pi_1(M)$, then the action is semiconjugate to $\rho : \Gamma \to Aut(M)$.

Ingredients: (1) $\Gamma \to G$ is quasiisometric embedding (Lubutzig-Mozus-Raghunathan)
(2) G/Γ has small cusps
(3) The weights of any G-representation that sends some element in γ to hyperbolic element are never poportional to the roots.

Theorem B: $\mathbb{Z}^n$ acts on M, contains an anosov element γ_0 and no rank-1 factor, then the action is smoothly conjugate to a linear action.

Ingredients: (1) Uniform exponential mixing.
(2) W^s, W^u, W^{ss} etc. of the γ_0 action are nilpotent.

Remark (Prasad-Raghunathan) For typical $\gamma \in \Gamma$, $C_\Gamma(\gamma)$ contains a $\mathbb{Z}^{rank(G)}$.

Want to apply (B): need to show that if an γ_0 acts anosov, most γ should be so either.

Theorem (Katok-Leuis-Zimmer) $\Gamma = SL(n, \mathbb{Z})$, $M = \mathbb{T}^n$, ρ is the standard linear action, and there is a absolutely continuous invariant measure μ, then the above is true.

Proof: Zimmer's cocycle rigidity: for a.e. x, there is H s.t. $H^{-1}_{\alpha\gamma x} D_x \alpha(\gamma) H_x = \rho(\gamma)$, $\forall \gamma$. Apply this on a chosen set of conjugates of γ_0, and show that H can be extended to $\mathbb{T}^n$.

General case: the semiconjugate in theorem A induces a correspondence between γ-invariant measure μ and the invariant measure of the linear representation δ. The latter is classified by Benoist-Quint, hence use the previous theorem.