MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jhilburn@uoregon.edu

Speaker's Name: Alexander Braverman

Talk Title: Local L-functions and perverse sheaves on certain loop spaces

Date: 11 / 18 / 2014 **Time:** 2 : 00 **am / pm (circle one)**

List 6-12 key words for the talk: L-function, p-adic, perverse sheaves, loop space, reductive group, unipotent representation

Please summarize the lecture in 5 or fewer sentences:
Braverman gave a conjectural construction of L-factors for representations of a reductive group over a local non-archimedian field of positive characteristic. This construction avoids the local Langlands correspondence and is verified for representations containing an Iwahori fixed vector.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Braverman — Local L-factors and perverse sheaves on certain loop spaces.

Joint with Kazhdan + Bezrukavnikov

Goal: To give a "uniform" definition of local L-factors for repn of p-adic

Theorem Get the correct definition for repn with an Iwahori fixed vector.

Immediate goal Extend to all unipotent reps

Setup

G reductive $\qquad \rho : G^\vee \to GL(V)$

$\sigma : G \to G_m$ character $\qquad \rho \circ \sigma^\vee =$ sum of positive chars of G_m

K local non archimedean field, $\quad \pi \in irr(G(K))$

$L(\pi, \rho, \sigma, s) = \dfrac{1}{P(q^{-s})}$ $\qquad P =$ polynomial

$p(t) = 1 +$ higher order ...

Let W_K be Weil-Deligne group

$W_K = G_a \rtimes Weil_K$.

$\cup$

I_K $\qquad W_K / I_K \simeq \mathbb{Z} \ni Fr$

Local Langlands

$Irr\, G \longrightarrow Hom(W_K, G^\vee) / \text{conjugacy}$

Given $\rho: G^\vee \to GL(V)$, $\tau: W_K \to G^\vee$

L-function
$$\frac{1}{\det\left(1 - \sigma^\vee(q^{-s})\, \rho\circ\tau(\mathrm{Fr})\right)\Big|_{V^I}}$$

$q = \#$ elements of residue field.

want to construct fns without Local Langlands.

Ex

$G = GL(n)$

$\sigma = \det$

$\rho =$ standard

Godement–Jacquet procedure

($n=1$ Tate's thesis)

start with $\pi \in \mathrm{Irr}\, GL(n,K)$ form integrals:

choose c_π – matrix coefficient of π

$\varphi \in S(\mathrm{Mat}(n,K))$

locally constant compactly supported.

$$Z(\varphi, c_\pi, s) = \int_{GL(n,K)} \varphi(g)\, c_\pi(g)\, |\det(g)|^s\, dg$$

Theorem 1) $Z(\varphi, c_\pi, s)$ is a rational function of q^s

2) for given π the space of all $Z(\varphi, c_\pi, s)$ forms a fractional ideal of $\mathbb{C}[q^s, q^{-s}]$

3) $\exists$ generator $L(\pi, s) = \dfrac{1}{1 + \text{higher order in } q^{-s}}$

Non-trivial part

$S(\mathrm{Mat}(n, K))$ — bimodule over $G(K)$

$\cup\!|$ $\Updownarrow$

$\mathcal{H}_G$ bimodule over

$\mathcal{H}_G$ = Hecke alg

$L(\pi, s)$ "measures the difference"

In general we need a space $Sp(G) \supseteq \mathcal{H}_G$

bimodule over $\mathcal{H}_G$

Remark $\Gamma \subset G(K)$ open compact subgroup

1) assume that $\pi^\Gamma \neq 0$. Then

$Sp(G)^{\Gamma \times \Gamma}$ is sufficient

2) $\Gamma = G(\mathcal{O})$ $\mathcal{H}(G)^{\Gamma \times \Gamma} \underset{\eta}{\overset{\text{Satake}}{\simeq}} K_0(\mathrm{Rep}\, G^\vee)$

$$S_\rho(G)^{G(\mathcal{O}) \times G(\mathcal{O})} \simeq \eta^{-1}\left(\mathrm{Sym}^*(\rho)\right) * \mathcal{H}(G)^{G(\mathcal{O}) \times G(\mathcal{O})}$$

$$\mathrm{Sym}^*(\rho) = \bigoplus_{n \geq 0} \mathrm{Sym}^n(\rho)$$

$\mathrm{supp}\left(\eta^{-1}(\mathrm{Sym}^n(\rho))\right)$ are disjoint

3) $\Gamma = I$ – Iwahori

$$\mathcal{H}(G)^{I\times I} \underset{\substack{\text{K-L}\\ \text{Ginzburg}}}{\simeq} K_{G^\vee \times \mathbb{C}^*}(St_{G^\vee})$$

$$St\, G^\vee = \{(b_1, b_2, x) \mid b_1, b_2 \text{ - borel subalgebras in } \mathfrak{g}^\vee,\ x\text{-nilpotent } x \in b_1 \cap b_2\}$$

$\cap$

$$St_{G^\vee, \rho} = \{(b_1, b_2, x, v) \mid v \in V,\ \rho(x)(v) = 0\}$$

Claim $\quad S_\rho(G)^{I\times I} = K_{G^\vee \times \mathbb{C}^*}(St_{G^\vee, \rho})$

Idea of the definition

Relations between choices of ρ and choices of $G = G^* \cap G_+$ affine normal semigroup

G = invertible elements

Given $G_+ \rightsquigarrow \rho$ - rep of $G^\vee$

Ex. $\dim G/[G,G] = 1$ for any irreducible ρ will appear in this way.

ex 1. $G = GL(n) \subseteq Mat(n)$

corresponds to ρ = st rep of $G^\vee = GL(n)$

2. Choose $r > 0$

$G = GL(2)$ if r odd

$SL(2) \times G_m$ $\quad r$ is even

$\rho = \mathrm{Sym}^r k^2$

$$G_+ = \left\{(A, x) \;\middle|\; \begin{array}{l} A \in Mat(2) \\ x \in G_m \\ \det A = x^r \end{array}\right\}$$

singular when $r > 1$

Step 2

ρ comes from G_t

Naive Idea $\qquad S_\rho = S(G_t(K))$ compactly supported

Conjecture 1, $\quad S_\rho(G) =$ Space of global sections of some sheaf on $G_t(K)$

char $k > 0 \qquad K = \mathbb{F}_q((t))$

$G(K)$ ind scheme over $\mathbb{F}_q$

1) Choice of G_t defines another ind-scheme structure on $G(K)$.

2) Consider all possible perverse ℓ-adic ~~sheaves~~ sheaves on this new ind ~~scheme~~ scheme.

3) Corresponding functions given by the Frobenius They span $S_\rho(G)$.

X_t affine variety $/K \qquad K = k((t))$

$X_t \supset X$ open subset

$X_t(K)$ has natural structure of ind-scheme $/k$

$X_t(K) \supset X(K) \simeq X_t(K)_0$ also ind-scheme

If X is also affine then $X_t(K)_0 \neq X(K)$

$X(K) \to X_t(K_0)$ locally closed embedding.

$X_+ = \mathbb{A}^1$ $\quad$ $X_+(K)$ connected.

$X = \mathbb{G}_m$ $\quad$ $X(K)$ $\mathbb{Z}$-many connected components

Theorem 1) Given $A \in \mathrm{Perv}(G(K))$ can construct a function on $G(K)$ which should correspond to GM extension of A to $G_+(K)_0$.

$\forall n \geq 0 \rightsquigarrow A_n$ on $G(K)$

$\to$ functions $\sum_{n \geq 0} \chi(A_n)$

(long + hard) 2) If we define $S_p(G)$ as the span of these functions then $S_p I \times I = K_{G \times G}(St_{G^\vee, p})$