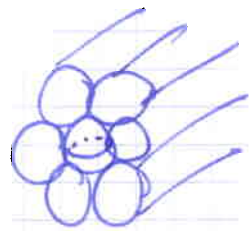
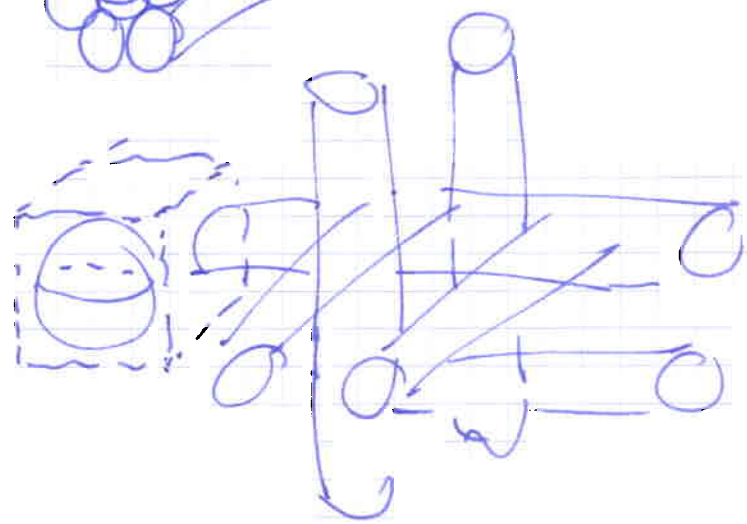
Kuperberg

How many infinite unit cylinders can be tangent to a sphere w/o overlap?

six is possible:



or



is six best possible?

Almost confirmed. Twice. Appel & Szabó 1991 $\theta(3) \leq 8$
Brass & Wenk 2000 $\theta(3) \leq 7$

Def: $\theta(n)$ = max # of unit cylinders that touch the unit ball in $\mathbb{R}^n$ in a packing (w/o interior intersection)

Asymptotics :

$$N(n-1) \le \theta(n) \le N(n)$$

Newton / kissing number. = maximal number of neighbors in a packing.

dual problem to kissing number:

minimal number of neighbors in a covering (Hadwiger number) $H(n) = n+1$.

Def: $\varphi(n)$ = minimal # of open unit cylinders that can cover the closed unit ball in $\mathbb{R}^n$

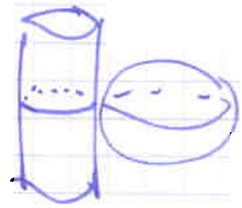
Prop: $\varphi(n) = \left\lceil \frac{n+1}{2} \right\rceil$

Pf: "$\le$": $n+1$ open balls can cover closed ball. pair balls to be contained in $\left\lceil \frac{n+1}{2} \right\rceil$ cylinders.

"$\ge$": k lines in $\mathbb{R}^n$ are contained in flat of $\le 2k-1$ dimension.
If $k \le \left\lceil \frac{n+1}{2} \right\rceil - 1$, then $2k-1 \le n-1$, so cannot cover closed ball.

Modify primal problem:

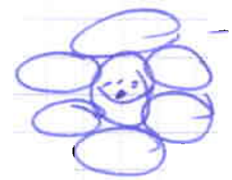
attach finite cylinders in the middle



long enough ⇝ $\Theta(n)$

how long is long enough?

2 is not long enough.



can attach
8 cylinders.

Beer question: is $2+\varepsilon$ long enough?

Paco: no. rotate north/south pole cylinders
relative to the six equator ones to make room.